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Response to “Comment on ‘Variational approach to the volume viscosity of fluids’” [Phys. Fluids 18, 109101 (2006)]

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We respond to the Comment of Markus Scholle and therewith revise our material entropy constraint to account for the production of entropy. © 2006 American Institute of Physics.

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The following is our Response to the Comment by M. Scholle on the subject article.¹

Inconsistency No. 1: We agree with a reviewer of Scholle’s comment and reiterate his reply:

“For a steady flow, α is independent of time at a given position but not necessarily constant with position as the convective part of the material derivative is $\mathbf{v} \cdot \nabla \alpha = -A$. I do not see that integrating this in time implies linear growth in α .”

In other words, in steady flow explicit time derivatives $\partial/\partial t$ are zero, in which case integration over time is not meaningful.

Inconsistency No. 2: We agree that the assumption of negligibly small entropy production, as expressed in our Eq. (28), which we call the “acoustic approximation,” restricts the analysis to a small subclass of flows. We take this opportunity to generalize the analysis for the case where entropy production is not negligibly small. We again assume a single dissipative process. Unless otherwise indicated, equation numbers (28) and above refer to those of our original article; equation numbers in the present Response begin again with (1).

To include entropy production, we replace Eq. (28) with^{2,3}

$$\frac{DS}{Dt} - \frac{L}{T} A^2 = 0. \quad (1)$$

This replacement will not affect the result of the variational procedure [Eq. (32)], because A is neither an independent variable nor a coordinate of the variation. Upon substitution of Eq. (8) (in the original paper) into (1) the entropy production becomes

$$\frac{DS}{Dt} = \frac{1}{LT} \left(\frac{D\xi}{Dt} \right)^2. \quad (2)$$

However, the entropy production term appearing in the dynamic equation of state [differentiating Eq. (41) with respect to time],

$$\frac{D\rho}{Dt} = \left(\frac{\partial \rho}{\partial P} \right)_{\xi S} \frac{DP}{Dt} + \left(\frac{\partial \rho}{\partial \xi} \right)_{PS} \frac{D\xi}{Dt} + \left(\frac{\partial \rho}{\partial S} \right)_{P\xi} \frac{DS}{Dt}, \quad (3)$$

as well as in the equation of motion [Eq. (39)],

$$\frac{D\mathbf{v}}{Dt} = -\nabla \Omega - \frac{1}{\rho} \nabla P + \alpha \nabla \frac{D\xi}{Dt} + \beta \nabla \frac{DS}{Dt}, \quad (4)$$

cannot be ignored under these conditions. Upon substituting Eq. (2) into (3) and (4) we find

$$\frac{D\rho}{Dt} = \left(\frac{\partial \rho}{\partial P} \right)_{\xi S} \frac{DP}{Dt} + \left(\frac{\partial \rho}{\partial \xi} \right)_{PS} \frac{D\xi}{Dt} + \left(\frac{\partial \rho}{\partial S} \right)_{P\xi} \frac{1}{LT} \left(\frac{D\xi}{Dt} \right)^2, \quad (5)$$

$$\frac{D\mathbf{v}}{Dt} = -\nabla \Omega - \frac{1}{\rho} \nabla P + \alpha \nabla \frac{D\xi}{Dt} + \frac{\beta}{LT} \nabla \left(\frac{D\xi}{Dt} \right)^2. \quad (6)$$

We can eliminate the progress variable ξ from the equation of motion [Eq. (6)] by letting

$$\alpha = -\frac{C}{\rho} \left(\frac{\partial \rho}{\partial \xi} \right)_{PS}, \quad (7)$$

and

$$\beta = -\frac{C}{\rho} \left(\frac{\partial \rho}{\partial S} \right)_{P\xi}, \quad (8)$$

where C is a constant. Compatibility with the equations of acoustical propagation requires that

$$C = \frac{\tau_{VS}}{\kappa_S^\infty} = \frac{\tau_{PS}}{\kappa_S^0} = \tau_{PS} \rho_0 a_0^2. \quad (9)$$

Upon using

$$\rho \kappa_S^\infty = \left(\frac{\partial \rho}{\partial P} \right)_{\xi S} \quad \text{and} \quad \frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{v}$$

and comparing with the last two terms of (5), we obtain

$$\begin{aligned} \alpha \frac{D\xi}{Dt} + \frac{\beta}{LT} \left(\frac{D\xi}{Dt} \right)^2 &= -\frac{C}{\rho} \left[\frac{D\rho}{Dt} - \left(\frac{\partial \rho}{\partial P} \right)_{\xi S} \frac{DP}{Dt} \right] \\ &= \tau_{PS} \rho_0 a_0^2 \nabla \cdot \mathbf{v} + \tau_{VS} \frac{DP}{Dt}. \end{aligned} \quad (10)$$

When we substitute (10) into (6), we recover our original

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volume viscosity terms in the equation of motion:

$$\rho \frac{D\mathbf{v}}{Dt} = -\rho \nabla \Omega - \nabla P + \nabla \left[\tau_{PS} \rho_0 a_0^2 \nabla \cdot \mathbf{v} + \tau_{VS} \frac{DP}{Dt} \right], \quad (11)$$

and consequently the same modified Navier-Stokes equations (45) or (46) as before. We conclude that the production of entropy by internal processes, no matter how large, has no effect upon the macroscopic motion of a fluid. This is not a

surprising result, for the entropy production in viscous shear flow likewise is not manifest in the Navier-Stokes equation.

We thank Professor Scholle for his Comment.

¹A. J. Zuckerwar and R. L. Ash, "Variational approach to the volume viscosity of fluids," Phys. Fluids **18**, 047101 (2006).

²H. J. Bauer, "Phenomenological theory of the relaxation phenomena in gases," in *Physical Acoustics IIA*, edited by W. P. Mason (Academic, New York, 1965).

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